

Shock acoustical waves on linear and non-linear dielectric and flexoelectric materials

A. E. Giannakopoulos

National Technical University of Athens, Greece



Intersonic shear cracks and fault ruptures

ARES J. ROSAKIS*

Aeronautics and Mechanical Engineering, California Institute of Technology,
Pasadena, CA 91125, USA

$$\sin \theta = \frac{c_s}{V}$$

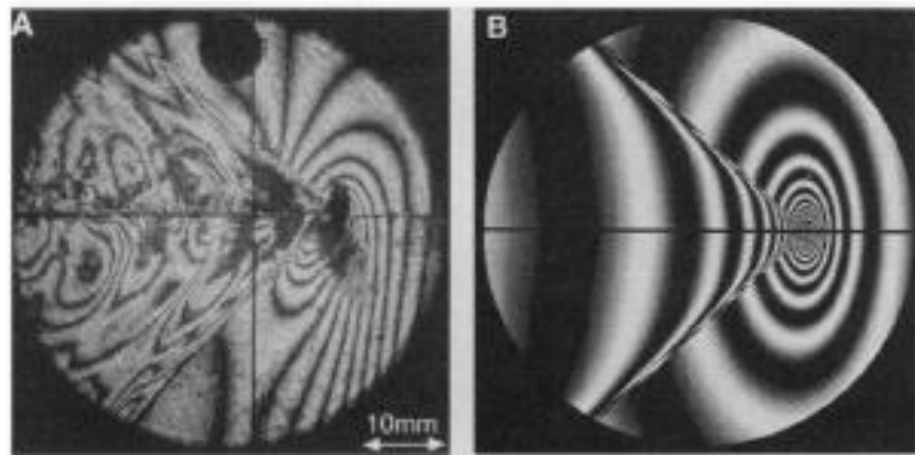
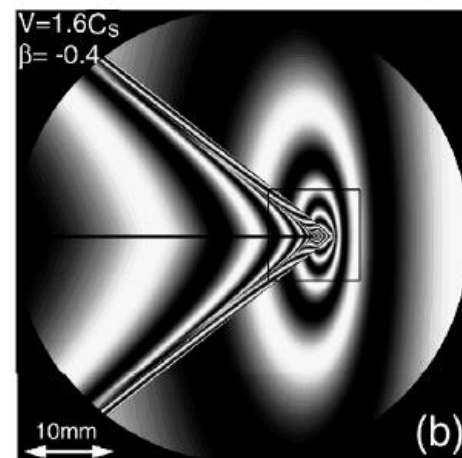
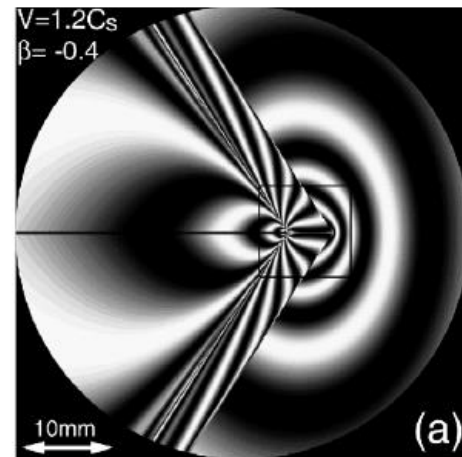
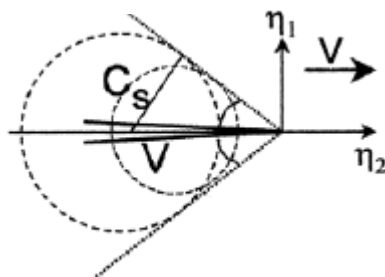
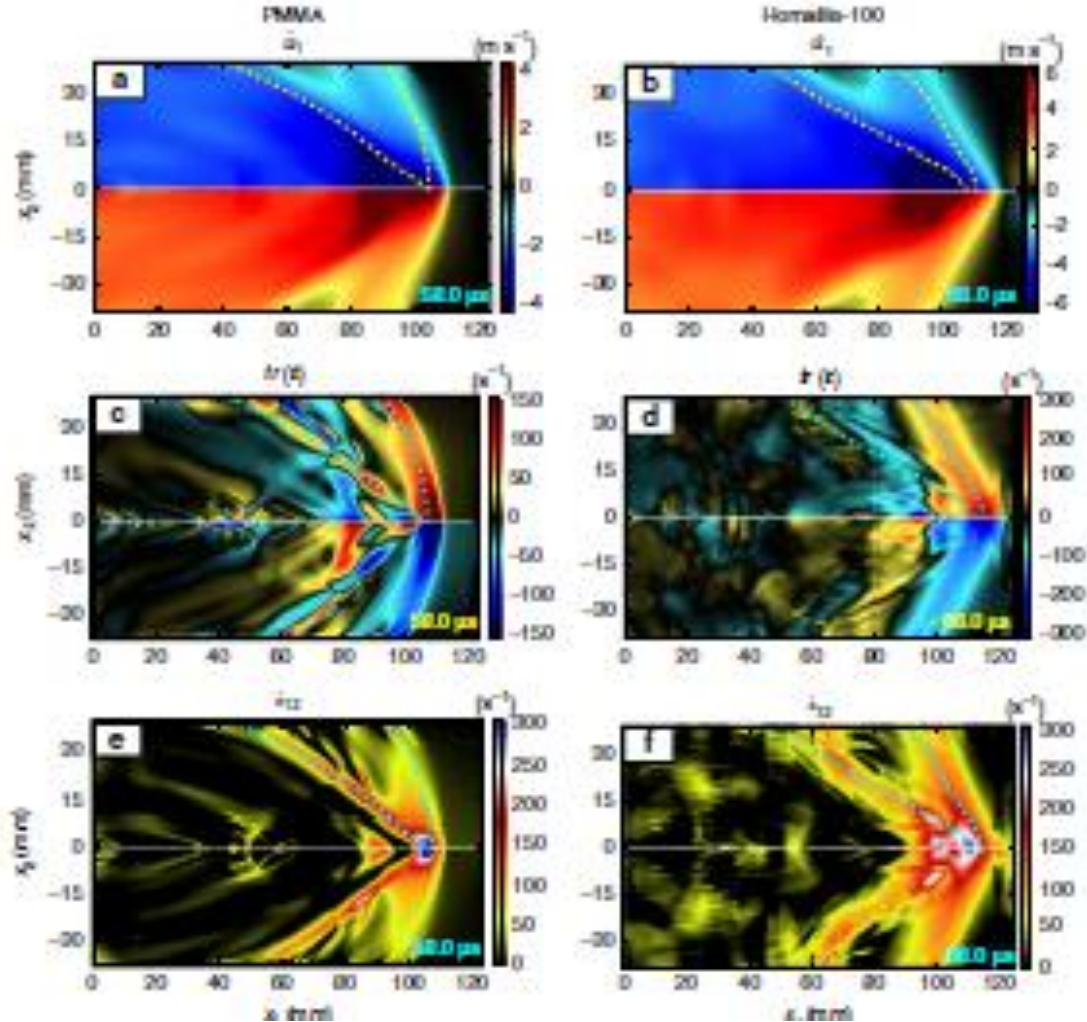
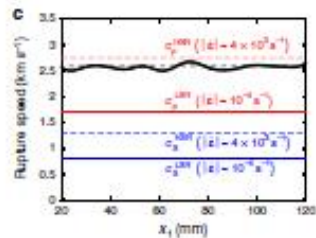
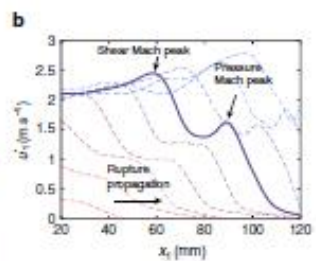
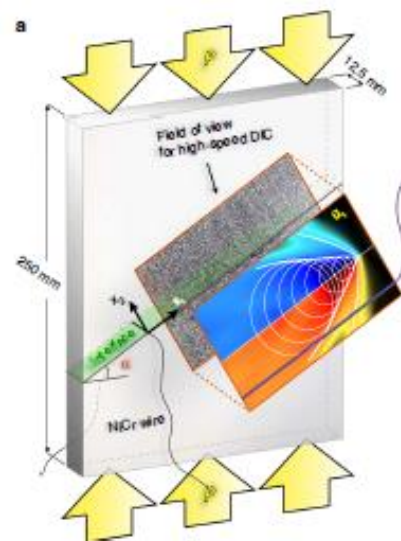


Fig. 4. Enlarged view of the isochromatic fringe pattern around a steady-state mode II intersonic crack along a weak plane in Homalite-100. (A) Experimental pattern. (B) Freund's theoretical prediction [13]. For both cases, $\beta = 43^\circ$ and $v = 1.47c_s$.

Figure 21. Synthetic isochromatic patterns constructed using the steady state velocity weakening cohesive zone model of reference [58].



Lattice wave emission from a moving dislocation

H. Koizumi

Department of Physics, Meiji University, Tama-ku, Kawasaki 214-8571, Japan

H. O. K. Kirchner

Institut de Sciences des Matériaux, Université Paris-Sud, Bâtiment 413, F-91405 Orsay, France

T. Suzuki

Institute of Industrial Science, University of Tokyo, Roppongi, Minato-ku, Tokyo 106-8558, Japan

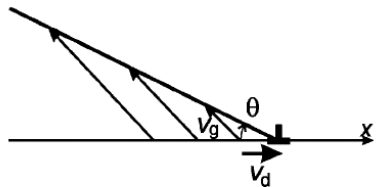


FIG. 15. A dislocation moving with velocity v_d emits lattice waves that propagate with group velocity v_g . The successively emitted waves with a certain \mathbf{k} vector leave a line behind the dislocation. The angle between the line and the $-x$ direction is $\theta = \tan^{-1}[v_{gy}/(v_d - v_{gx})]$.

$$\sin \theta = \frac{\ell \sqrt{6}}{H} \frac{c_s}{V}$$

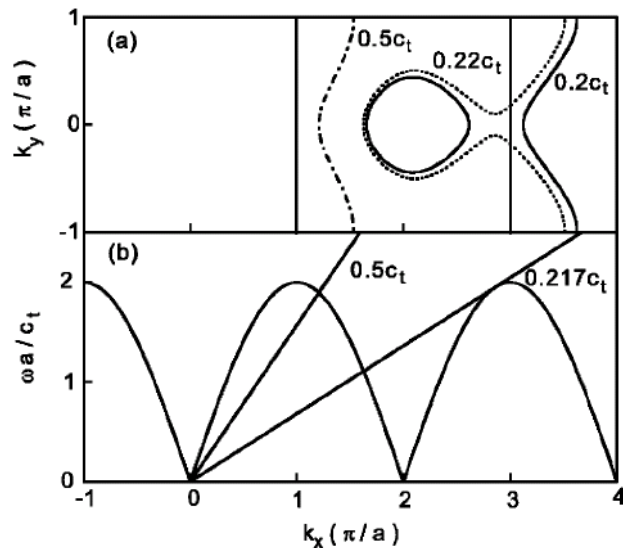


FIG. 13. (a) Wave vector \mathbf{k} that satisfies $\omega(\mathbf{k}_x, \mathbf{k}_y) = \mathbf{k} \cdot \mathbf{v}_d$ and (b) phonon-dispersion curve along the axis $k_y=0$, $\omega(k_x, 0)$ for the square lattice.

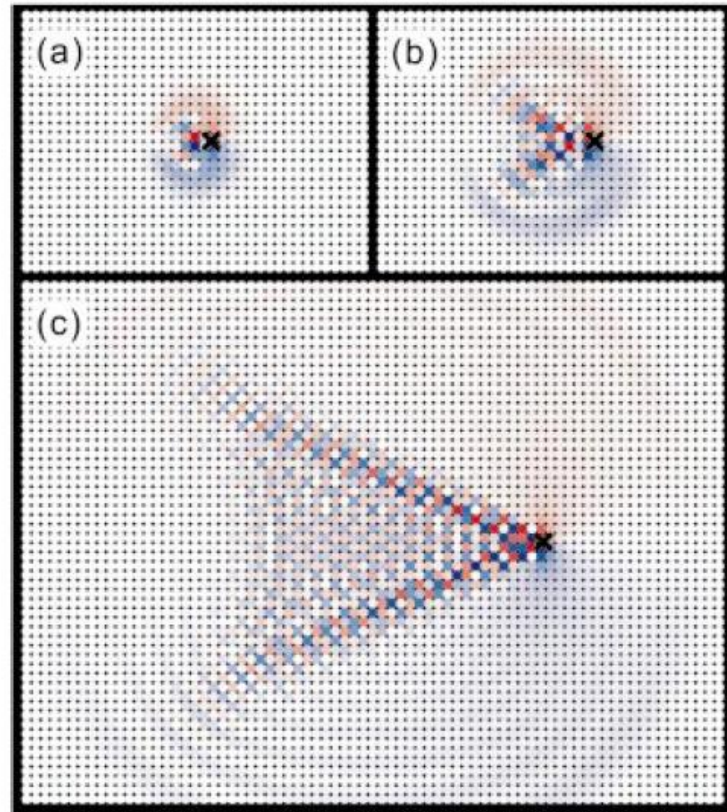


FIG. 4. (Color) Emitted waves from a moving dislocation with a velocity of $0.51c_t$ under the applied stress $F=0.051G$. (a) $t = 6.36a/c_t$, (b) $t = 6.95a/c_t$, (c) $t = 37.69a/c_t$. The meaning of the colors and the contrast are the same as in Fig. 3. The maximum velocity v_{\max} of the atomic rows is $0.125c_t$, $0.176c_t$, and $0.201c_t$ in the cases (a), (b), and (c), respectively. The position of the dislocation is indicated with a cross.

Compressional and shear wakes in a two-dimensional dusty plasma crystal

V. Nosenko,* J. Goree,† and Z. W. Ma

Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242, USA

D. H. E. Dubin

Department of Physics, University of California at San Diego, La Jolla, California 92093, USA

A. Piel

Institut für Experimentelle und Angewandte Physik, Christian-Albrechts Universität, Kiel, Germany

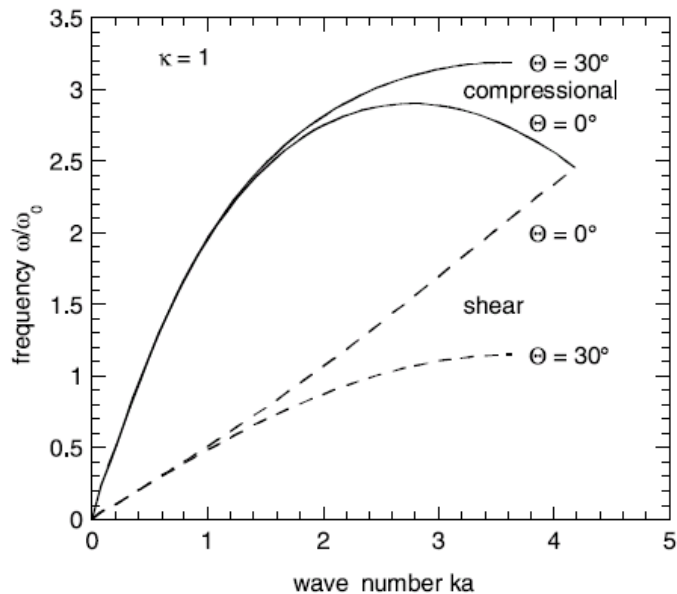
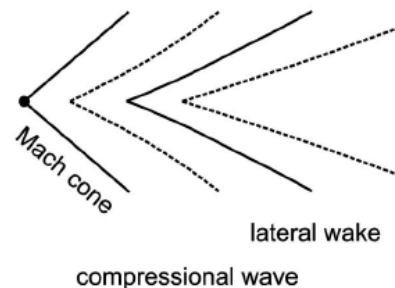
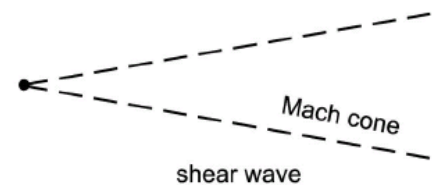
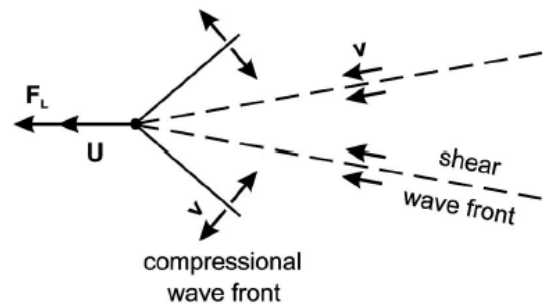
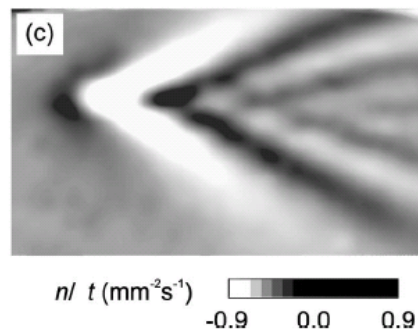
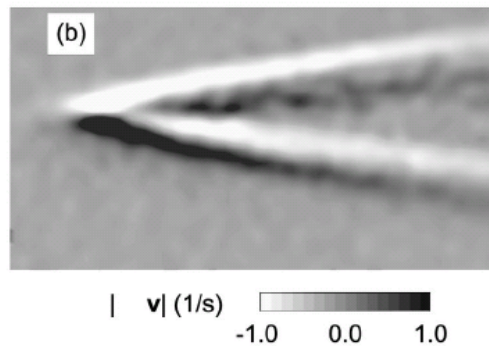
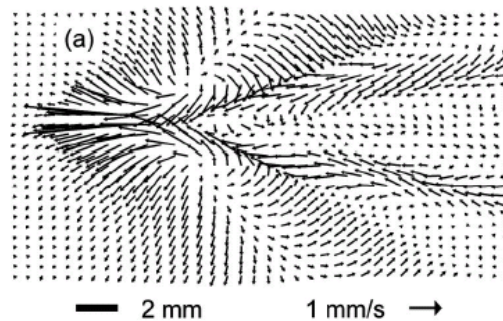


FIG. 3. Theoretical dispersion relation for a 2D triangular Yukawa lattice. It has two modes, compressional and shear. The shear wave has less dispersion, i.e., ω is more nearly $\propto k$ over a wide range of k than the compressional wave. The wave's propagation direction Θ is measured with respect to the primitive vector of the lattice. Reproduced from Fig. 3 of Ref. [12].



Motivation – Outline:

- Seismology
- Activation of Faults
- Fracture Mechanics
- Dielectricity combined with flexoelectricity
- Elimination of polarization
- Dynamic anti-plane problem
- Dispersion relation
- Theoretical and FEM analysis
- Nonlinearities and soliton waves

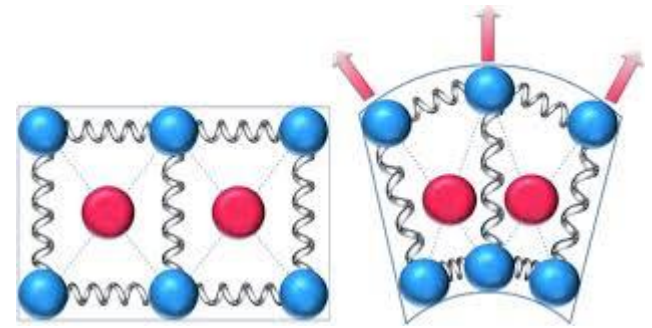
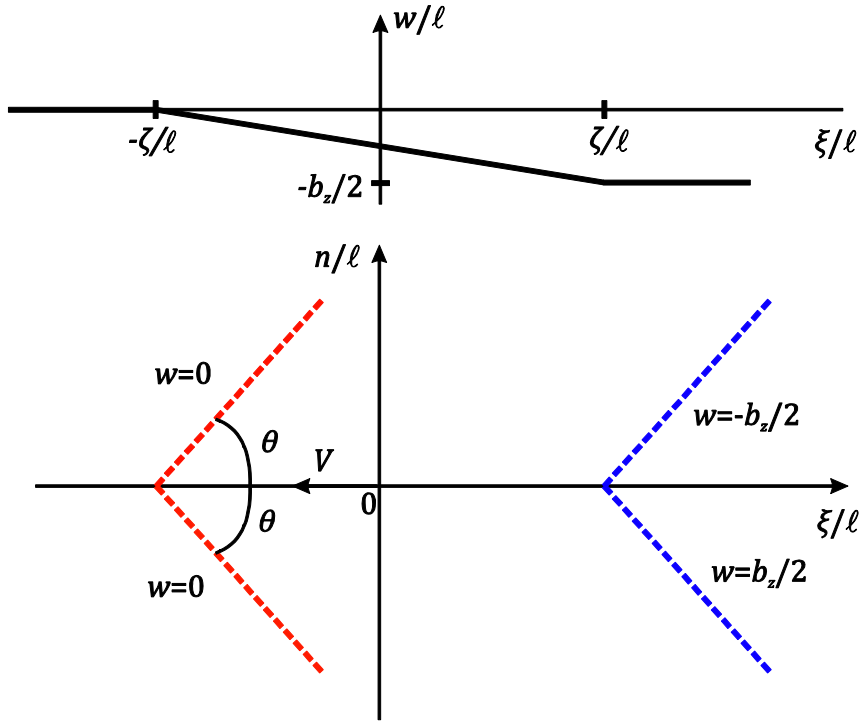


Illustration of induced polarization due to non-uniform bending deformation of a centro-symmetric (non-piezoelectric) material. [Krichen and Sharma \(2016\)](#)

Mach lines



$$\sin \theta = \frac{c_s}{V}$$

$$w = f(\Xi) = f\left(\xi \pm \eta \sqrt{\frac{V^2}{c_s^2} - 1}\right)$$

$$\sin \theta = \frac{\ell \sqrt{6} c_s}{H V}$$

$$\bar{\eta} = \frac{\xi}{\ell} \pm \frac{\eta}{\ell} \sqrt{\frac{H^2 V^2}{6 \ell^2 c_s^2} - 1}$$

Isotropic anti-plane dielectric flexoelectric:

$$\beta = 0$$

Assume: $X_z = 0$, $\Phi_{,z} = 0$, $E_z^0 = 0$

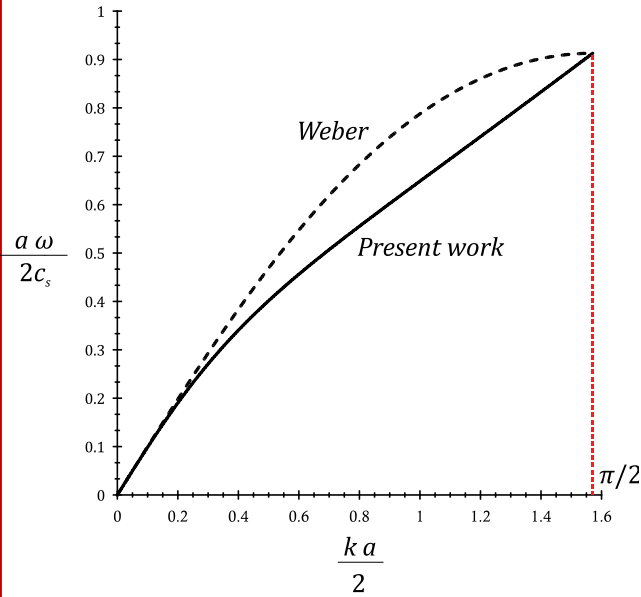
$$\mu \nabla^2 u_z - \underbrace{\left(\frac{\mu(b_{44} + b_{77})}{a} - \frac{(e_{44} - f_{12})^2}{a} \right)}_{\frac{\mu \ell^2}{2}} \nabla^4 u_z = \rho \ddot{u}_z - \underbrace{\frac{(b_{44} + b_{77})}{a}}_{\frac{1}{12} H^2} \rho \nabla^2 \ddot{u}_z$$

$$\mu > 0, a > 0, f_{44} > 0, e_{44} > 0, b_{44} + b_{77} > 0, \mu(b_{44} + b_{77}) - e_{44}^2 > 0$$

$$\frac{\ell^2}{2} = \frac{(b_{44} + b_{77}) - (e_{44} + f_{12})^2 / \mu}{a} > 0$$

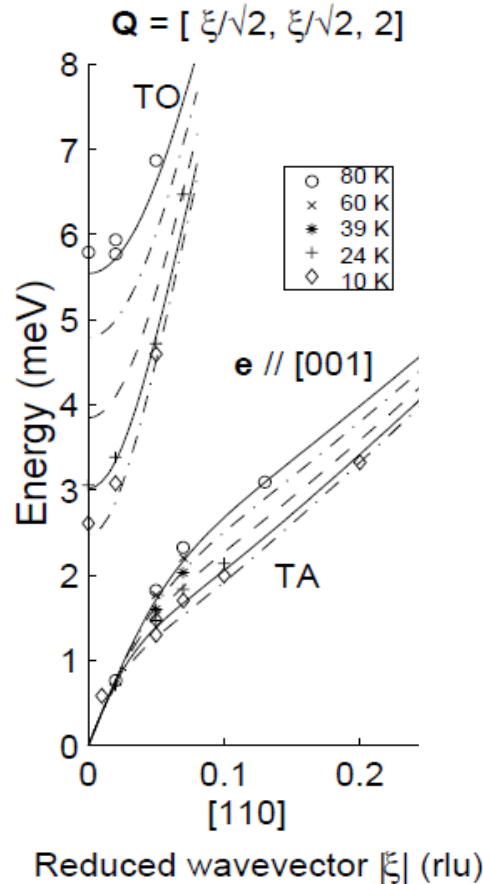
$$\frac{H^2}{12} = \frac{(b_{44} + b_{77})}{a} > 0$$

Dispersion relation:



$$k \rightarrow 0, \quad \frac{d\omega}{dk} = c_s$$

$$k \rightarrow \infty, \quad \frac{d\omega}{dk} = c_s \frac{\ell\sqrt{6}}{H}$$



$$\frac{\omega}{k} = c_s \left(\frac{1 + \frac{\ell^2}{2} k^2}{1 + \frac{H^2}{12} k^2} \right)^{1/2}$$

$$a^2 \omega^2 = c_s^2 \frac{\sin^2(ka/2)}{1 + 2(f'/f) \sin^2(ka/2)}$$

$$H/(\ell\sqrt{6}) = 1.938 \quad \longrightarrow \quad V/c_s \approx 0.67$$

$a/2 = \ell/\sqrt{2}$: Atomic dimension

$0 \leq ka/2 \leq \pi/2$: Wave number

$f'/f = 0.1$: ionic/bond forces

Isotropic anti-plane dielectric flexoelectric:

$$\beta = 0$$

Typical material constants (Maraghandi et al., 2006):

$$c_{44} = \mu = 0.325 \cdot 10^{-2} \left[\frac{\text{dyne}}{\text{nm}^2} \right]$$

$$e_{44} = 0.356 \cdot 10^{15} \left[\frac{\text{dyne nm}}{\text{C}} \right]$$

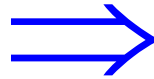
$$f_{12} = 0.01125 \cdot 10^7 \left[\frac{\text{dyne nm}}{\text{C}} \right]$$

$$b_{44} = 0.5255 \cdot 10^{32} \left[\frac{\text{dyne nm}^4}{\text{C}^2} \right]$$

$$b_{77} = 1.921 \cdot 10^{32} \left[\frac{\text{dyne nm}^4}{\text{C}^2} \right]$$

$$a = 8.767 \cdot 10^{33} \left[\frac{\text{dyne nm}^2}{\text{C}^2} \right]$$

$$\varepsilon_0 = 8.854 \cdot 10^{-35} \left[\frac{\text{C}^2}{\text{dyne nm}^2} \right]$$

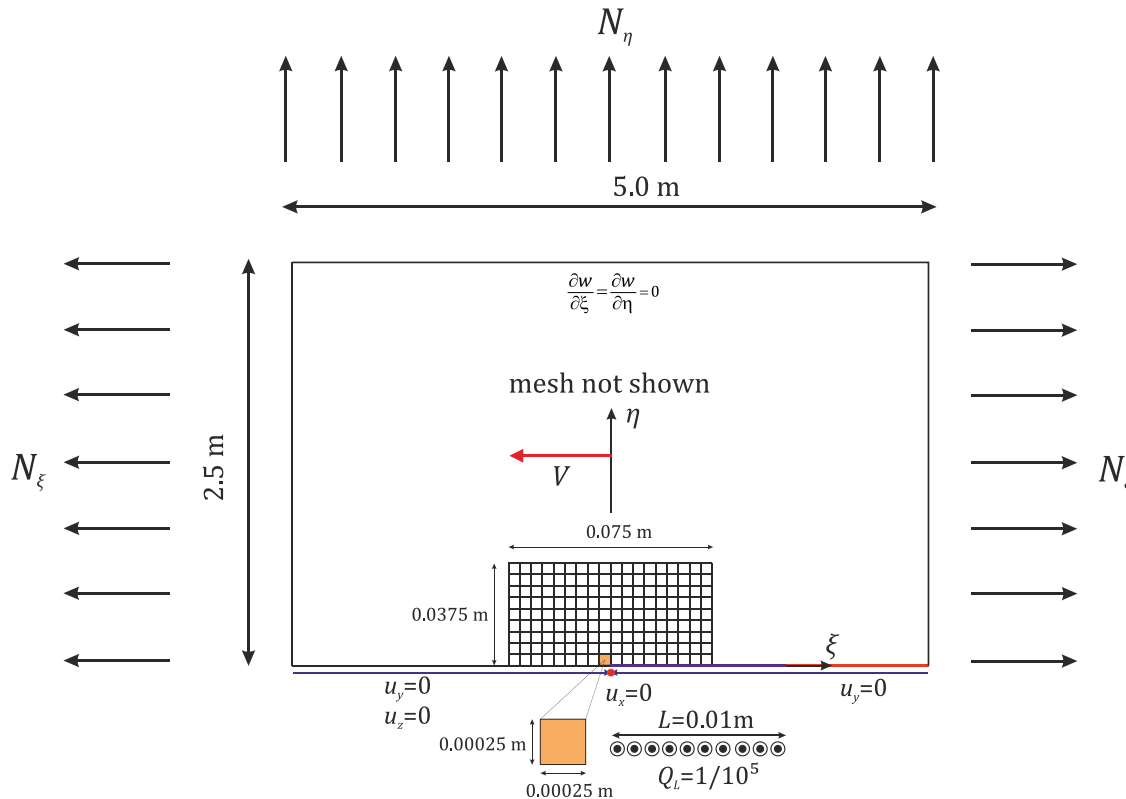


$$\frac{\ell}{\sqrt{2}} = \sqrt{\frac{(b_{44} + b_{77})}{a} - \frac{(e_{44} - f_{12})^2}{\mu a}} = 0.1532 \text{ nm}$$
$$\frac{H}{\sqrt{12}} = \sqrt{\frac{12(b_{44} + b_{77})}{a}} = 0.1671 \text{ nm}$$

$$\rho = 5.3176 \left[\frac{\text{g}}{\text{cm}^3} \right]$$

$$\text{strain gradient length} = 0.049 - 0.14 \text{ [nm]}$$

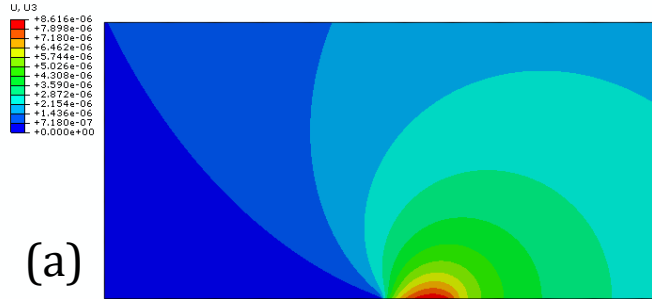
Isotropic anti-plane dielectric flexoelectric: FEM model



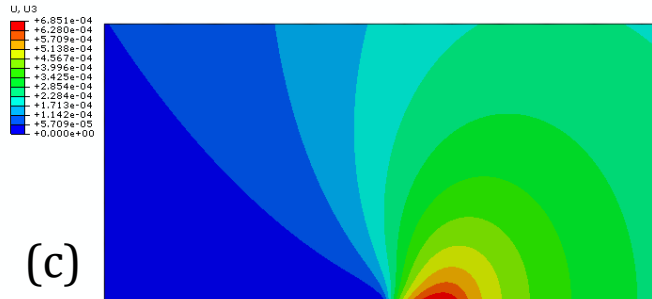
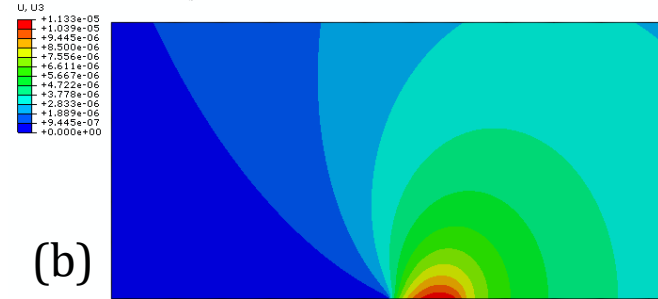
- We model the anti-plane space as a thin plate which extends multiple times the characteristic lengths of the problem.
- The plate's Poisson ratio is set to zero and orthotropic elasticity was tuned to capture the micro-inertia length.
- Pre-stretching was applied according to the crack velocity and the micro-structural length of the problem.
- Only half the space is discretized by 100000 linear quadrilateral elements (S4R in ABAQUS notation) and 600 linear triangular elements (S3 in ABAQUS notation).

Contour fields of out of plane displacements: Influence of velocity

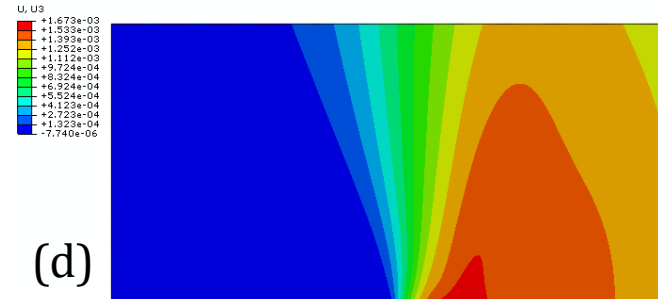
$$(V/c_s)^2 = 0, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0$$



$$(V/c_s)^2 = 0.5, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0$$



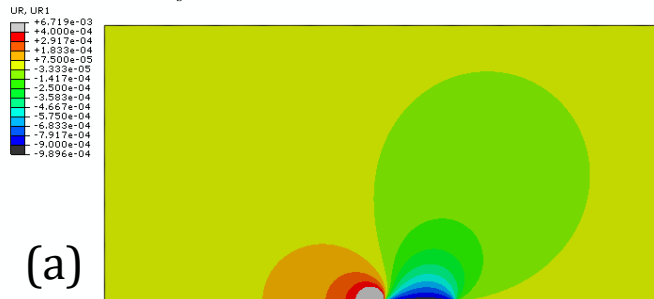
$$(V/c_s)^2 = 0.5, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0.003$$



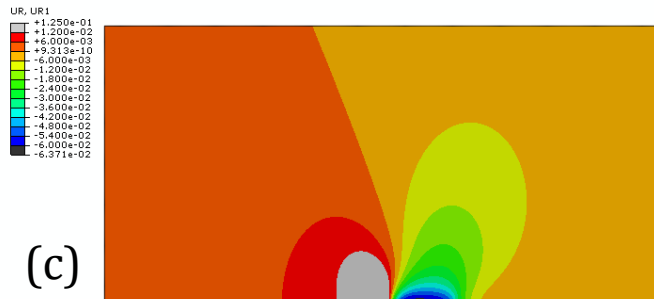
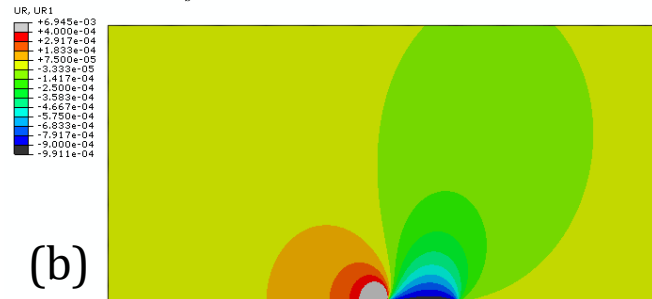
$$(V/c_s)^2 = 0.95, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0.003$$

Contour fields of rotations $w_{,2}$: Influence of velocity

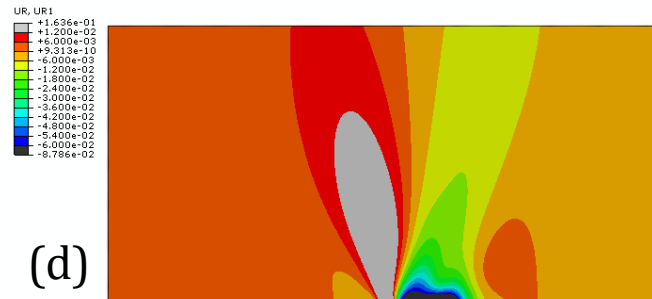
$$(V/c_s)^2 = 0, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0$$



$$(V/c_s)^2 = 0.5, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0$$



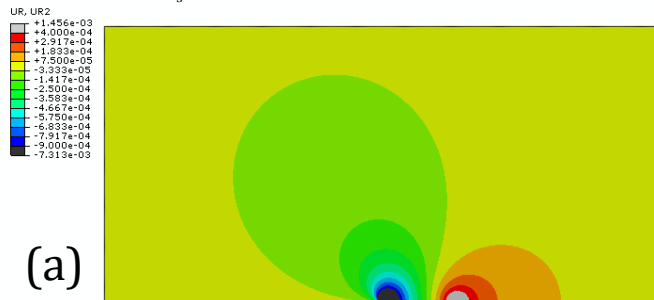
$$(V/c_s)^2 = 0.5, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0.003$$



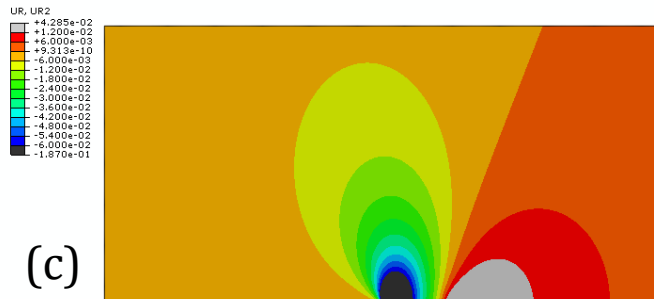
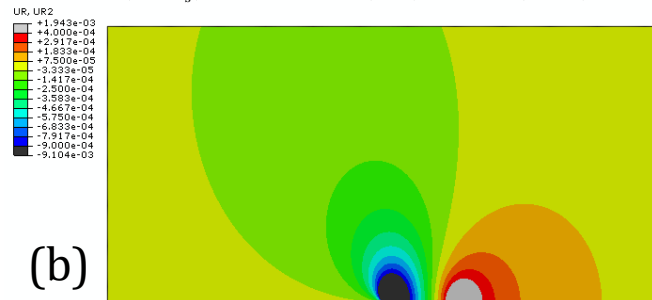
$$(V/c_s)^2 = 0.95, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0.003$$

Contour fields of rotations $w_{,1}$: Influence of velocity

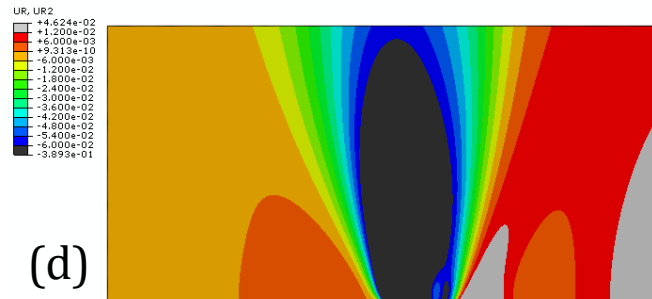
$$(V/c_s)^2 = 0, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0$$



$$(V/c_s)^2 = 0.5, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0$$



$$(V/c_s)^2 = 0.5, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0.003$$



$$(V/c_s)^2 = 0.95, H^2/(6\ell^2) = 0, \ell/(L\sqrt{2}) = 0.003$$

Conclusions:

- Antiplane problems in flexoelectricity
- Couple stress (micro-structural length, micro-inertia length)
- Prestressed plate analogue and FEM methodology
- Steady state results
- Dispersion analysis
- Fracture faults
- Anharmonic elasticity
- Viscosity
- Solitons